Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$. Also find the associated radius of convergence.

1) $f(x) = \cos x$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad R = \infty$$

2) $f(x) = \sin 2x$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}, \quad R = \infty$$

3) $f(x) = (1+x)^{-3}$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)(n+1) x^n}{2}, \quad R = 1$$

4) $f(x) = xe^x$  $\boxed{\sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}, \quad R = \infty}$

Find the Taylor series for $f(x)$ centered at the given value of $a$. Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$. Also find the associated radius of convergence.

5) $f(x) = 1 + x + x^2, \quad a = 2$  $\boxed{7 + 5(x-2) + (x-2)^2, \quad R = \infty}$

6) $f(x) = e^x, \quad a = 3$  $\boxed{\sum_{n=0}^{\infty} \frac{e^3}{n!}(x-3)^n, \quad R = \infty}$

7) $f(x) = \sin x, \quad a = \dfrac{\pi}{2}$

$$\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{(x-\pi/2)^{2n}}{(2n)!}, \quad R = \infty}$$

8) $f(x) = \dfrac{1}{\sqrt{x}}, \quad a = 9$

$$\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n \cdot 3^{2n+1} \cdot n!} (x-9)^n, \quad R = 9}$$

Use a derived Maclaurin series to obtain the Maclaurin series for the given function. Also find the associated radius of convergence.

9) $f(x) = e^{-x/2}$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^n, \quad R = \infty}$$

10) $f(x) = x\tan^{-1} x$ $\boxed{\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{2n+1}, \quad R=1}$

11) $f(x) = x\cos 2x$ $\boxed{\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{(2n)!}x^{2n+1}, \quad R=\infty}$

12) $f(x) = \sin^2 x$ [Hint: Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$.] $\boxed{\sum_{n=1}^{\infty}\frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}, \quad R=\infty}$

13) $f(x)=\begin{cases} \dfrac{x-\sin x}{x^3} & \text{if } x\neq 0 \\ \dfrac{1}{6} & \text{if } x=0 \end{cases}$

$$\boxed{\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+3)!}, \quad R=\infty}$$

Evaluate the indefinite integral as an infinite series.

14) $\int x\cos(x^3)\ dx$

$$\boxed{C+\sum_{n=0}^{\infty}(-1)^n\frac{x^{6n+2}}{(6n+2)(2n)!}}$$

15) $\int \frac{\sin x}{x}\ dx$

$$\boxed{C+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}}$$

16) $\int \frac{e^x - 1}{x}\, dx$  $\boxed{C + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}}$

Use series to evaluate the limit.

17) $\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$  $\boxed{\frac{1}{3}}$

18) $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$  $\boxed{\frac{1}{120}}$

Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for each function.

19) $y = e^{-x^2}\cos x$ $\boxed{1 - \frac{3}{2}x^2 + \frac{25}{24}x^4 + \cdots}$

20) $y = \frac{x}{\sin x}$ $\boxed{1 + \frac{1}{6}x^2 + \frac{7}{360}x^4 + \cdots}$

Find the sum of the series.

21) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{n!}$ $\boxed{e^{-x^4}}$

22) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$ $\boxed{\dfrac{\sqrt{3}}{2}}$

23) $1 - \ln 2 + \dfrac{(\ln 2)^2}{2!} - \dfrac{(\ln 2)^3}{3!} + \cdots$ $\boxed{\dfrac{1}{2}}$