Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

1) $r = 2\sin\theta, \quad \theta = \frac{\pi}{6}$ $\boxed{\sqrt{3}}$

2) $r = 2 - \sin\theta, \quad \theta = \frac{\pi}{3}$ $\boxed{\frac{2-\sqrt{3}}{1-2\sqrt{3}}}$

3) $r = 1 + \cos\theta, \quad \theta = \frac{\pi}{6}$ $\boxed{-1}$

Find the points on the given curve where the tangent line is horizontal or vertical.

4) $r = 3\cos\theta$

Horizontal: $\left(\frac{3}{\sqrt{2}}, \frac{\pi}{4}\right)$, $\left(-\frac{3}{\sqrt{2}}, \frac{3\pi}{4}\right)$ Vertical: $(3,0)$, $\left(0, \frac{\pi}{2}\right)$

5) $r = 1 + \cos\theta$

Horizontal: $\left(\frac{3}{2}, \frac{\pi}{3}\right)$, $(0,\pi)$, $\left(\frac{3}{2}, \frac{5\pi}{3}\right)$ Vertical: $(2,0)$, $\left(\frac{1}{2}, \frac{2\pi}{3}\right)$, $\left(\frac{1}{2}, \frac{4\pi}{3}\right)$

6) $r^2 = \sin 2\theta$

Horizontal: $\left(\sqrt[4]{\frac{3}{4}}, \frac{\pi}{3}\right)$, $\left(\sqrt[4]{\frac{3}{4}}, \frac{4\pi}{3}\right)$, $(0,0)$ Vertical: $\left(\sqrt[4]{\frac{3}{4}}, \frac{\pi}{6}\right)$, $\left(\sqrt[4]{\frac{3}{4}}, \frac{7\pi}{6}\right)$, $(0,0)$

Find the area of the region that is bounded by the given curve and lies in the specified sector.

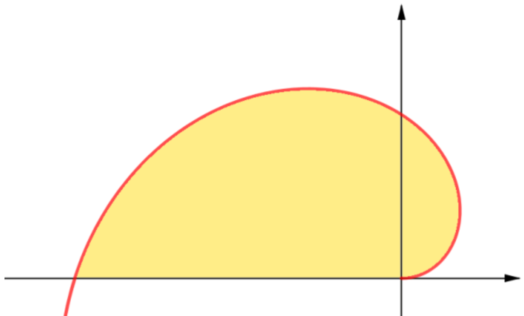
7) $r = \sqrt{\theta}, \quad 0 \le \theta \le \frac{\pi}{4}$ $\boxed{\frac{1}{64}\pi^2}$

8) $r = \sin\theta, \quad \frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$ $\boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{8}}$

9) $r = \sqrt{\sin\theta}, \quad 0 \le \theta \le \pi$ $\boxed{1}$

Find the area of the shaded region.

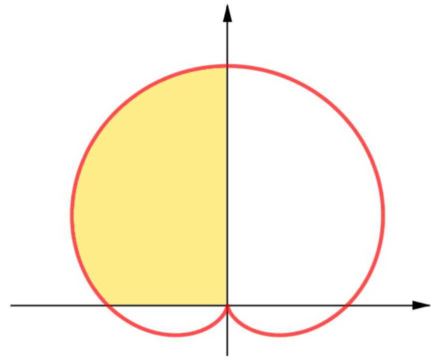
10)



$r = \theta$

$$\boxed{\frac{1}{6}\pi^3}$$

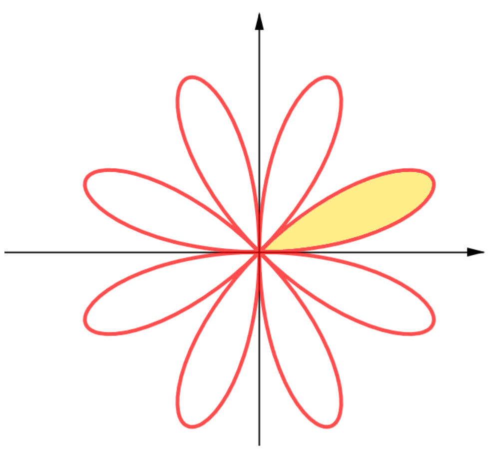
11)



$r = 1 + sin\theta$

$$\boxed{\frac{3\pi}{8}+1}$$

12)

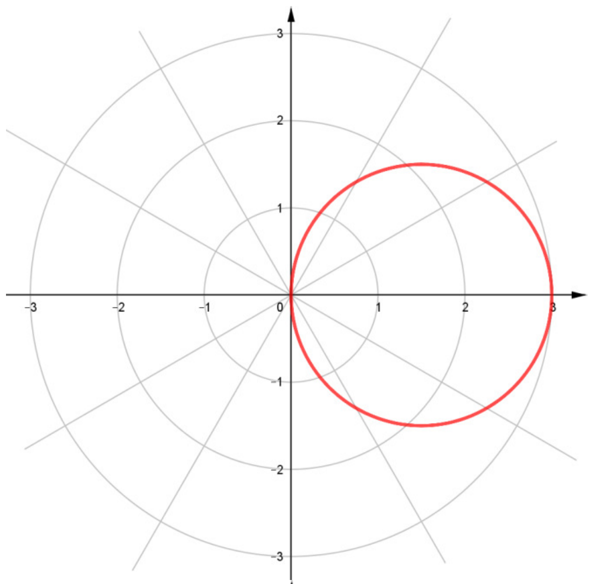


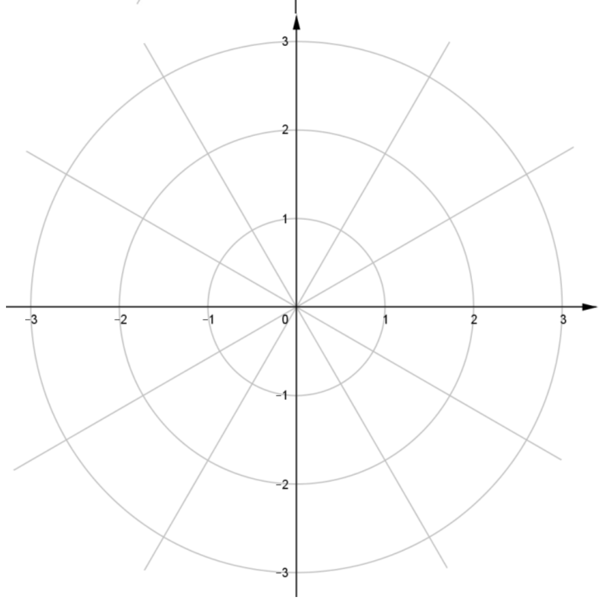
$r = sin(4\theta)$

$$\boxed{\frac{\pi}{16}}$$

Sketch the curve and find the area that it encloses.

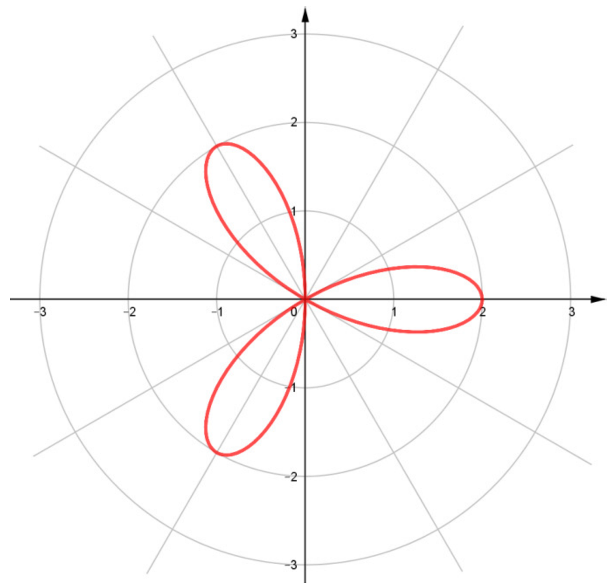
13) $r = 3\cos\theta$

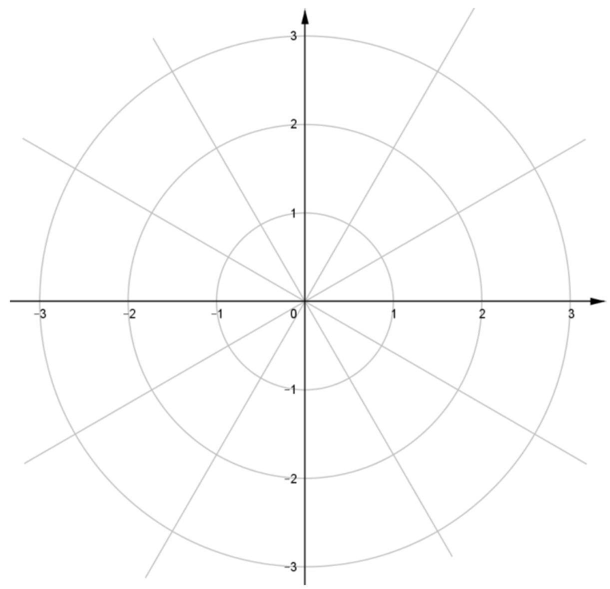




$\boxed{\dfrac{9\pi}{4}}$

14) $r = 2\cos 3\theta$





$\boxed{\pi}$

Find the area of the region enclosed by one loop of the curve.

15) $r = 1 + 2\sin\theta$ (inner loop) $\boxed{\pi - \frac{3\sqrt{3}}{2}}$

16) Find the area enclosed by the loop of the **strophoid**: $r = 2\cos\theta - \sec\theta$ $\boxed{2 - \frac{\pi}{2}}$

Find the area of the region that lies inside the first curve and outside the second curve.

17) $r = 4\sin\theta,\quad r = 2$ $\boxed{\frac{4\pi}{3} + 2\sqrt{3}}$

18) $r^2 = 8\cos 2\theta,\quad r = 2$ $\boxed{4\sqrt{3} - \frac{4\pi}{3}}$

19) $r = 2 + \sin\theta, \quad r = 3\sin\theta$ $\boxed{\frac{9\pi}{4}}$

20) $r = 3\cos\theta, \quad r = 1 + \cos\theta$ $\boxed{\pi}$

Find the area of the region that lies inside both curves.

21) $r = \sin\theta, \quad r = \cos\theta$ $\boxed{\frac{\pi}{8} - \frac{1}{4}}$

22) $r = \sin 2\theta, \quad r = \cos 2\theta$ $\boxed{\frac{\pi}{2} - 1}$

23) $r^2 = 2\sin 2\theta, \quad r = 1$ $\boxed{2-\sqrt{3}+\frac{\pi}{3}}$

Find all points of intersection of the given curves.

24) $r = \sin\theta, \quad r = \cos\theta$ $\boxed{(0,0), \left(0,\frac{\pi}{2}\right), \left(\frac{1}{\sqrt{2}},\frac{\pi}{4}\right)}$

25) $r = 2, \quad r = 2\cos 2\theta$ $\boxed{(2,0), \left(2,\frac{\pi}{2}\right), (2,\pi), \left(2,\frac{3\pi}{2}\right)}$

26) $r = \sin\theta, \quad r = \sin 2\theta$ $\boxed{(0,0), \left(\frac{\sqrt{3}}{2},\frac{\pi}{3}\right), \left(\frac{\sqrt{3}}{2},\frac{2\pi}{3}\right)}$

27) Use a graphing device to estimate the values of θ for which the cures $r = 3 + \sin 5\theta$ and $r = 6\sin\theta$ intersect. Then estimate the area that lies inside both curves.

$$\boxed{\theta \approx 0.58 \text{ and } 2.57 \quad A \approx 10.41}$$

Find the exact length of the polar curve.

28) $r = 3\sin\theta, \quad 0 \le \theta \le \dfrac{\pi}{3}$ $\boxed{\pi}$

29) $r = e^{2\theta}, \quad 0 \le \theta \le 2\pi$ $\boxed{\dfrac{\sqrt{5}}{2}\left(e^{4\pi} - 1\right)}$

Use a calculator to find the length of the curve correct to four decimal places.

30) $r = 3\sin 2\theta$ $\boxed{\approx 29.0653}$

31) $r = 1 + \cos(\theta/3)$ $\boxed{\approx 19.6676}$