

Find $\frac{dy}{dx}$.

1) $x = t - t^3$, $y = 2 - 5t$

$$\frac{-5}{1-3t^2}$$

Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

2) $x = t^4 + 1$, $y = t^3 + t$, $t = -1$

$$y = -x$$

3) $x = e^{\sqrt{t}}$, $y = t - \ln t^2$, $t = 1$

$$y = -\frac{2}{e}x + 3$$

4) $x = \cos \theta + \sin 2\theta, \quad y = \sin \theta + \cos 2\theta, \quad t = 0$

$$y = \frac{1}{2}x + \frac{1}{2}$$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward?

5) $x = 4 + t^2, \quad y = t^2 + t^3$

$$\frac{dy}{dx} = 1 + \frac{3}{2}t, \quad \frac{d^2y}{dx^2} = \frac{3}{4t}, \quad \text{CU: } t > 0$$

6) $x = t - e^t, \quad y = t + e^{-t}$

$$\frac{dy}{dx} = -e^{-t}, \quad \frac{d^2y}{dx^2} = \frac{e^{-t}}{1 - e^t}, \quad \text{CU: } t < 0$$

7) $x = 2\sin t, \quad y = 3\cos t, \quad 0 < t < 2\pi$

$$\frac{dy}{dx} = -\frac{3}{2}\tan t, \quad \frac{d^2y}{dx^2} = -\frac{3}{4}\sec^3 t, \quad \text{CU: } \frac{\pi}{2} < t < \frac{3\pi}{2}$$

Find the points on the curve where the tangent is horizontal or vertical.

8) $x = 2t^3 + 3t^2 - 12t, \quad y = 2t^3 + 3t^2 + 1$

Horizontal: (0,1) and (13,2)
Vertical: (20,-3) and (-7,6)

9) $x = 2\cos \theta, \quad y = \sin 2\theta$

Horizontal: $(\pm\sqrt{2}, \pm 1)$ (4 points)
Vertical: $(\pm 2, 0)$

10) At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with equations $x = -7t$, $y = 12t - 5$?

$$\boxed{(-5, 6) \text{ or } \left(-\frac{208}{27}, \frac{32}{3}\right)}$$

11) Use the parametric equations of an ellipse, $x = a \cos \theta$, $y = b \sin \theta$, $0 \leq \theta \leq 2\pi$, to find the area that it encloses.

$$\boxed{\pi ab}$$

12) Find the area bounded by the curve $x = \cos t$, $y = e^t$, $0 \leq t \leq 2\pi$, and the lines $y = 1$ and $x = 0$.

$$\boxed{\frac{1}{2}(e^{\pi/2} - 1)}$$

Set up, but do not evaluate, an integral that represents the length of the curve.

13) $x = t - t^2$, $y = \frac{4}{3}t^{3/2}$, $1 \leq t \leq 2$

$$\boxed{L = \int_1^2 \sqrt{1 + 4t^2} dt}$$

14) $x = \ln t$, $y = \sqrt{t+1}$, $1 \leq t \leq 5$

$$\boxed{L = \int_1^5 \frac{t+2}{2t\sqrt{t+1}} dt}$$

Find the length of the curve.

$$15) x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1 \quad \boxed{2(2\sqrt{2} - 1)}$$

$$16) x = e^t \cos t, \quad y = e^t \sin t, \quad 0 \leq t \leq \pi \quad \boxed{\sqrt{2}(e^\pi - 1)}$$

$$17) x = e^t - t, \quad y = 4e^{t/2}, \quad -8 \leq t \leq 3 \quad \boxed{e^3 - e^{-8} + 11}$$