Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$

1) $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$, S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$, oriented upward.

$$\boxed{0}$$

2) $\mathbf{F}(x, y, z) = x^2 y^3 z\,\mathbf{i} + \sin(xyz)\,\mathbf{j} + xyz\,\mathbf{k}$, S is the part of the cone $y^2 = x^2 + z^2$ that lies between the planes $y = 0$ and $y = 3$, oriented in the direction of the positive y-axis.

$$\boxed{\frac{2187}{4}\pi}$$

3) $\mathbf{F}(x, y, z) = xyz\,\mathbf{i} + xy\,\mathbf{j} + x^2 yz\,\mathbf{k}$, S consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$, oriented outward [Hint: use $\iint_{S_1} \text{curl } \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r} = \iint_{S_2} \text{curl } \mathbf{F} \cdot d\mathbf{S}$]

$$\boxed{0}$$

Use Stokes' Theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$. In each case C is oriented counterclockwise as viewed from above.

4) $\mathbf{F}(x,y,z)=(x+y^2)\,\mathbf{i}+(y+z^2)\,\mathbf{j}+(z+x^2)\,\mathbf{k}$, C is the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

$$\boxed{-1}$$

5) $\mathbf{F}(x,y,z)=e^{-x}\,\mathbf{i}+e^{x}\,\mathbf{j}+e^{z}\,\mathbf{k}$, C is the boundary of the part of the plane $2x+y+2z=2$ in the first octant.

$$\boxed{2e-4}$$

6) $\mathbf{F}(x,y,z)=yz\,\mathbf{i}+2xz\,\mathbf{j}+e^{xy}\,\mathbf{k}$, C is the circle $x^2+y^2=16,\ z=5$.

$$\boxed{80\pi}$$

7) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x^2 z\,\mathbf{i} + xy^2\,\mathbf{j} + z^2\,\mathbf{k}$ and C is the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 9$ oriented counterclockwise as viewed from above.

$$\boxed{\frac{81\pi}{2}}$$

8) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x^2 y\,\mathbf{i} + \frac{1}{3}x^3\,\mathbf{j} + xy\,\mathbf{k}$ and C is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$ oriented counterclockwise as viewed from above.

$$\boxed{\pi}$$

9) Verify Stokes' Theorem is true for the vector field $\mathbf{F}(x, y, z) = y^2\,\mathbf{i} + x\,\mathbf{j} + z^2\,\mathbf{k}$, S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upward.

$$\boxed{\pi}$$