

1) Find the first partial derivatives $f_x(x, y)$ and $f_y(x, y)$ of the function.

a) $f(x, y) = x^2 - 2y^2 + 4$

h) $z = \ln \frac{x+y}{x-y}$

b) $f(x, y) = \frac{4x^3}{y^2}$

i) $z = \frac{xy}{x^2 + y^2}$

c) $f(x, y) = 2y^2\sqrt{x}$

j) $h(x, y) = e^{-(x^2+y^2)}$

d) $f(x, y) = y^3 - 2xy^2 - 1$

k) $z = \cos xy$

l) $z = \sin 5x \cos 5y$

e) $z = e^{x/y}$

m) $z = e^y \sin xy$

f) $z = ye^{y/x}$

n) $f(x, y) = \int_x^y (t^2 - 1) dt$

g) $z = \ln \sqrt{xy}$

2) Use the limit definition of partial derivatives to find $f_x(x, y)$ and $f_y(x, y)$.

a) $f(x, y) = 3x + 2y$

b) $f(x, y) = \frac{1}{x+y}$

3) Evaluate f_x and f_y at the given point.

a) $f(x, y) = \sin xy, \left(2, \frac{\pi}{4}\right)$

b) $f(x, y) = \frac{2xy}{\sqrt{4x^2 + 5y^2}}, (1, 1)$

4) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

a) $x^2 + y^2 + z^2 = 3xyz$

b) $x - z = \arctan(yz)$

5) Find the first partial derivatives with respect to x , y and z .

a) $f(x, y, z) = 3x^2y - 5xyz + 10yz^2$

b) $w = \frac{7xz}{x + y}$

6) Evaluate f_x , f_y , and f_z at the given point.

a) $f(x, y, z) = \frac{xy}{x + y + z}, (3, 1, -1)$

b) $f(x, y, z) = z \sin(y + x), \left(0, \frac{\pi}{2}, -4\right)$

7) Show that the mixed partials $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$ are equal. (Verify that the conclusion of Clairaut's theorem holds.)

a) $z = x^4 - 3x^2y^2 + y^4$

b) $z = 2xe^y - 3ye^{-x}$

8) For $f(x, y)$, find all values of x and y such that $f_x(x, y) = 0$ and $f_y(x, y) = 0$ simultaneously.

a) $f(x, y) = x^2 + xy + y^2 - 2x + 2y$

b) $f(x, y) = e^{x^2+xy+y^2}$

9) Find the indicated partial derivative.

a) $f(r, s, t) = r \ln(rs^2t^3)$; f_{rss} , f_{rst}

b) $z = u\sqrt{v-w}$; $\frac{\partial^3 z}{\partial u \partial v \partial w}$

10) Show that the function $z = \cos(4x + 4ct)$ satisfies the wave equation $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$.