1) Use the definition of the limit of a function of two variables to verify the limit.

a) $\lim_{(x,y)\to(1,0)} x = 1$

b) $\lim_{(x,y)\to(a,b)} y = b$

2) Find the indicated limit by using the limits: $\lim_{(x,y)\to(a,b)} f(x,y) = 4$ and $\lim_{(x,y)\to(a,b)} g(x,y) = 3$

a) $\lim_{(x,y)\to(a,b)} \left[f(x,y) - g(x,y)\right]$

b) $\lim_{(x,y)\to(a,b)} \left[\dfrac{5f(x,y)}{g(x,y)}\right]$

c) $\lim_{(x,y)\to(a,b)} \left[\dfrac{f(x,y)+g(x,y)}{f(x,y)}\right]$

3) Find the limit and discuss the continuity of the function.

a) $\lim_{(x,y)\to(2,1)} 2x^2 + y$

b) $\lim_{(x,y)\to(2,4)} \dfrac{x+y}{x^2+1}$

c) $\lim_{(x,y)\to(-1,2)} \dfrac{x+y}{x-y}$

d) $\lim_{(x,y)\to(1,1)} \dfrac{x}{\sqrt{x+y}}$

e) $\lim_{(x,y)\to(0,1)} \dfrac{\arcsin xy}{1-xy}$

f) $\lim_{(x,y,z)\to(1,3,4)} \sqrt{x+y+z}$

4) Find the limit (if it exists). If the limit does not exist, explain why.

a) $\lim\limits_{(x,y)\to(1,-1)} \dfrac{x^2 y}{1+xy^2}$

b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{1}{x+y}$

c) $\lim\limits_{(x,y)\to(2,2)} \dfrac{x^2-y^2}{x-y}$

d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-y}{\sqrt{x}-\sqrt{y}}$

e) $\lim\limits_{(x,y)\to(2,1)} \dfrac{x-y-1}{\sqrt{x-y}-1}$

f) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+y}{x^2+y}$

g) $\lim\limits_{(x,y)\to(0,0)} \ln(x^2+y^2)$

h) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+xz}{x^2+y^2+z^2}$

i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+\sin^2 y}{2x^2+y^2}$

j) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$

k) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2 y}{x^4+y^2}$

l) $\lim\limits_{(x,y,z)\to(3,0,1)} e^{-xy} \sin\left(\dfrac{\pi z}{2}\right)$

5) Discuss the continuity of the functions f and g. Explain any differences.

$$f(x,y)=\begin{cases}\dfrac{x^4-y^4}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

$$g(x,y)=\begin{cases}\dfrac{x^4-y^4}{x^2+y^2} & (x,y)\neq(0,0)\\ 1 & (x,y)=(0,0)\end{cases}$$

6) Determine the set of appoints at which the function $f(x,y,z)=\dfrac{\sqrt{y}}{x^2-y^2+z^2}$ is continuous.

7) Use polar coordinate to find the limit. Let $x=r\cos\theta$ and $y=r\sin\theta$, note that $(x,y)\to(0,0)$ implies $r\to 0^+$.

a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^2}$

b) $\lim\limits_{(x,y)\to(0,0)} \cos(x^2+y^2)$

c) $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\cdot\ln(x^2+y^2)$ [use L'Hopital's Rule]

8) Use Spherical coordinates to find the limit. Let $x=\rho\sin\phi\cos\theta$, $y=\rho\sin\phi\sin\theta$, and $z=\rho\cos\phi$, note that $(x,y,z)\to(0,0,0)$ implies $\rho\to 0^+$

a) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^2+z^2}$

b) $\lim\limits_{(x,y,z)\to(0,0,0)} \tan^{-1}\left[\dfrac{1}{x^2+y^2+z^2}\right]$