

1) Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s).

a)  $f(x, y) = x^2 - y^2$ ,  $x^2 + y^2 = 1$

b)  $f(x, y, z) = 2x + 6y + 10z$ ,  $x^2 + y^2 + z^2 = 35$

c)  $f(x, y, z, t) = x + y + z + t$ ,  $x^2 + y^2 + z^2 + t^2 = 1$

a)  $f(\pm 1, 0) = 1$  Max,  $f(0, \pm 1) = -1$  Min

b)  $f(1, 3, 5) = 70$  Max,  $f(-1, -3, -5) = -70$  Min

c)  $f(0.5, 0.5, 0.5, 0.5) = 2$  Max,  $f(-0.5, -0.5, -0.5, -0.5) = -2$  Min

2) Find the extreme values of  $f(x, y) = e^{-xy}$  on the region  $x^2 + 4y^2 \leq 1$ .

$$f\left(\pm \frac{1}{\sqrt{2}}, \mp \frac{1}{2\sqrt{2}}\right) = e^{1/4} \text{ Max, } f\left(\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{2\sqrt{2}}\right) = e^{-1/4} \text{ Min}$$

- 3) Find the highest point on the curve of intersection of the cone  $x^2 + y^2 - z^2 = 0$  and the plane  $x + 2z = 4$ . [Hint: You are trying to maximize  $f(x, y, z) = z$ ].

$$z = 4, (-4, 0, 4)$$

- 4) The sum of the length and the girth (perimeter of a cross section) of a package carried by a delivery service cannot exceed 108 inches. Find the dimensions of the rectangular package of largest volume that may be sent.

$$36 \times 18 \times 18 \text{ inches}$$

- 5) Use Lagrange multipliers to find the dimensions of a right circular cylinder with volume  $V_0$  cubic units and minimum surface area.

$$\text{Dimensions: } r = \sqrt[3]{\frac{V_0}{2\pi}} \text{ and } h = 2\sqrt[3]{\frac{V_0}{2\pi}}$$