Verify that the divergence Theorem is true for the vector field $\mathbf{F}$ on the region E.

1) $\mathbf{F}(x, y, z) = 3x\,\mathbf{i} + xy\,\mathbf{j} + 2xz\,\mathbf{k}$, E is the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

$$\boxed{\frac{9}{2}}$$

2) $\mathbf{F}(x, y, z) = x^2\,\mathbf{i} + xy\,\mathbf{j} + z\,\mathbf{k}$, E is the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

$$\boxed{8\pi}$$

Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, that is, calculate the flux of $\mathbf{F}$ across S.

3) $\mathbf{F}(x, y, z) = e^x \sin y\,\mathbf{i} + e^x \cos y\,\mathbf{j} + yz^2\,\mathbf{k}$, S is the surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$.

$$\boxed{2}$$

4) $\mathbf{F}(x, y, z) = 3xy^2\,\mathbf{i} + xe^z\,\mathbf{j} + z^3\,\mathbf{k}$, S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

$$\boxed{\frac{9\pi}{2}}$$

5) $\mathbf{F}(x,y,z)=x^2y\,\mathbf{i}+xy^2\,\mathbf{j}+2xyz\,\mathbf{k}$, S is the surface of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$, and $x+2y+z=2$.

$$\boxed{\frac{2}{5}}$$

6) $\mathbf{F}(x,y,z)=\left(x^3+y\sin z\right)\mathbf{i}+\left(y^3+z\sin x\right)\mathbf{j}+3z\,\mathbf{k}$, S is the surface of the solid bounded by the hemispheres $z=\sqrt{4-x^2-y^2}$, $z=\sqrt{1-x^2-y^2}$ and the plane $z=0$.

$$\boxed{\frac{194}{5}\pi}$$