

1) Find the directional derivative of f at the given point in the direction indicated by the angle θ .

a) $f(x, y) = x^2y^3 - y^4$, $(2, 1)$, $\theta = \frac{\pi}{4}$

b) $f(x, y) = x \sin(xy)$, $(2, 0)$, $\theta = \frac{\pi}{3}$

a) $\boxed{6\sqrt{2}}$

b) $\boxed{2\sqrt{3}}$

2) Find the directional derivative of the function at the given point in the direction of the vector \vec{v} .

a) $f(x, y) = \ln(x^2 + y^2)$, $(2, 1)$, $\vec{v} = \langle -1, 2 \rangle$

b) $f(x, y, z) = \frac{x}{y+z}$, $(4, 1, 1)$, $\vec{v} = \langle 1, 2, 3 \rangle$

a) $\boxed{0}$

b) $\boxed{-\frac{9}{2\sqrt{14}}}$

3) Find the directional derivative of the function $g(x, y, z) = xye^z$ at $P(2, 4, 0)$ in the direction of $Q(0, 0, 0)$.

$$\boxed{-\frac{8}{\sqrt{5}}}$$

4) Given the function $f(x, y) = y \ln x$, $P(1, -3)$, and $\vec{u} = \left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle$ find the following:

a) The gradient of f .

b) The gradient at the point P .

c) The rate of change of f at P in the direction of the vector \vec{u} .

a) $\boxed{\left\langle \frac{y}{x}, \ln x \right\rangle}$

b) $\boxed{\langle -3, 0 \rangle}$

c) $\boxed{\frac{12}{5}}$

5) Find the maximum rate of change of f at the given point and the direction in which it occurs.

a) $f(x, y) = \frac{y^2}{x}, (2, 4)$

b) $f(x, y, z) = \tan(x + 2y + 3z), (-5, 1, 1)$

a) $4\sqrt{2}, \langle -4, 4 \rangle$

b) $\sqrt{14}, \langle 1, 2, 3 \rangle$

6) Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin xy$ at the point $(1, 0)$ has the value 1.

$$\theta = \frac{\pi}{2} \text{ or } 2\pi - \cos^{-1}\left(\frac{4}{5}\right)$$

7) Find all points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

$$\text{All points on } y = x + 1$$

8) Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$.

a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\vec{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

b) In which direction does V change most rapidly at P ?

c) What is the maximum rate of change at P ?

a) $\frac{32}{\sqrt{3}}$

b) $\langle 38, 6, 12 \rangle$

c) $2\sqrt{406}$

- 9) If $f(x, y) = x^2 + 4y^2$, find the gradient vector $\nabla f(2,1)$ and the use it to find the tangent line to the level curve $f(x, y) = 8$ at the point $(2,1)$.

$$\langle 4, 8 \rangle, x + 2y = 4$$

- 10) If $g(x, y) = x - y^2$, find the gradient vector $\nabla g(3,-1)$ and the use it to find the tangent line to the level curve $g(x, y) = 2$ at the point $(3,1)$.

$$\langle 1, 2 \rangle, x + 2y = 1$$