1) Find the directional derivative of f at the given point in the direction indicated by the angle θ.

a) $f(x, y) = x^2y^3 - y^4$, $(2,1)$, $\theta = \dfrac{\pi}{4}$

b) $f(x, y) = x\sin(xy)$, $(2,0)$, $\theta = \dfrac{\pi}{3}$

2) Find the directional derivative of the function at the given point in the direction of the vector $\vec{\mathbf{v}}$.

a) $f(x, y) = \ln(x^2 + y^2)$, $(2,1)$, $\vec{\mathbf{v}} = \langle -1, 2 \rangle$

b) $f(x, y, z) = \dfrac{x}{y+z}$, $(4,1,1)$, $\vec{\mathbf{v}} = \langle 1, 2, 3 \rangle$

3) Find the directional derivative of the function $g(x, y, z) = xye^z$ at $P(2,4,0)$ in the direction of $Q(0,0,0)$.

4) Given the function $f(x, y) = y\ln x$, $P(1,-3)$, and $\vec{\mathbf{u}} = \left\langle -\dfrac{4}{5}, \dfrac{3}{5} \right\rangle$ find the following:

a) The gradient of f.

b) The gradient at the point P.

c) The rate of change of f at P in the direction of the vector $\vec{\mathbf{u}}$.

5) Find the maximum rate of change of f at the given point and the direction in which it occurs.

a) $f(x,y)=\dfrac{y^2}{x}$, $(2,4)$

b) $f(x,y,z)=\tan(x+2y+3z)$, $(-5,1,1)$

6) Find the directions in which the directional derivative of $f(x,y)=x^2+\sin xy$ at the point $(1,0)$ has the value 1.

7) Find all points at which the direction of fastest change of the function $f(x,y)=x^2+y^2-2x-4y$ is $\mathbf{i}+\mathbf{j}$.

8) Suppose that over a certain region of space the electrical potential V is given by $V(x,y,z)=5x^2-3xy+xyz$.

a) Find the rate of change of the potential at $P(3,4,5)$ in the direction of the vector $\vec{\mathbf{v}}=\mathbf{i}+\mathbf{j}-\mathbf{k}$.

b) In which direction does V change most rapidly at P ?

c) What is the maximum rate of change at P ?

9) If $f(x,y) = x^2 + 4y^2$, find the gradient vector $\nabla f(2,1)$ and the use it to find the tangent line to the level curve $f(x,y) = 8$ at the point $(2,1)$.

10) If $g(x,y) = x - y^2$, find the gradient vector $\nabla g(3,-1)$ and the use it to find the tangent line to the level curve $g(x,y) = 2$ at the point $(3,1)$.