

1) Find the total differential.

a) $z = 2x^2y^3$

b) $z = x \cos y - y \cos x$

c) $w = 2z^3y \sin x$

2) If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, find the values of Δz and dz .

3) The radius r and height h of a right circular cylinder are measured with possible errors of 4% and 2%, respectively. Approximate the maximum possible percent error in measuring the volume.

4) A triangle is measured and two adjacent sides are found to be 3 inches and 4 inches long, with an included angle of $\frac{\pi}{4}$. The possible errors in measurement are 0.0625 inches for the sides and 0.02 radian for the angle. Approximate the maximum possible error in the computation of the area.

5) Show that the function $f(x, y) = x^2 - 2x + y$ is differentiable by finding values ε_1 and ε_2 as designated in the definition of differentiability, and verify that both ε_1 and $\varepsilon_2 \rightarrow 0$ as $(\Delta x, \Delta y) \rightarrow (0, 0)$.

6) Use the function below to show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist, but that f is not differentiable at $(0, 0)$.

$$f(x, y) = \begin{cases} \frac{3x^2y}{x^4 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

7) Find an equation of the tangent plane to the given surface at the specified point.

a) $z = 4x^2 - y^2 + 2y$; $(-1, 2, 4)$

b) $z = e^{x^2 - y^2}$; $(1, -1, 1)$

- 8) Show that that function is differentiable at the given point. Then find the lineararization $L(x, y)$ of the function at that point.
- a) $f(x, y) = x\sqrt{y}$; $(1, 4)$
 - b) $f(x, y) = \sin(2x + 3y)$; $(-3, 2)$
- 9) Find the linear approximation of the function $f(x, y) = \ln(x - 3y)$ at $(7, 2)$ and use it to approximate $f(6.9, 2.06)$.
- 10) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use it to approximate $f(3.02, 1.97, 5.99)$.