Find the $\text{curl}\,\vec{\mathbf{F}}$ for the vector field at the given point.

1) $\vec{\mathbf{F}}(x, y, z) = xyz\,\mathbf{i} + xyz\,\mathbf{j} + xyz\,\mathbf{k}$, at the point $(2,\ 1,\ 3)$.

$$\boxed{4\mathbf{i} - \mathbf{j} - 3\mathbf{k}}$$

2) $\vec{\mathbf{F}}(x, y, z) = e^{-xyz}\left(\mathbf{i} + \mathbf{j} + \mathbf{k}\right)$, at the point $(3,\ 2,\ 0)$.

$$\boxed{6\mathbf{i} - 6\mathbf{j}}$$

Find the $\text{div}\,\vec{\mathbf{F}}$ for the vector field at the given point.

3) $\vec{\mathbf{F}}(x, y, z) = e^{x}\sin y\,\mathbf{i} - e^{x}\cos y\,\mathbf{j} + z^{2}\,\mathbf{k}$, at the point $(3,\ 0,\ 0)$.

$$\boxed{0}$$

4) $\vec{\mathbf{F}}(x, y, z) = \ln(xyz)\left(\mathbf{i} + \mathbf{j} + \mathbf{k}\right)$, at the point $(3,\ 2,\ 1)$.

$$\boxed{\frac{11}{6}}$$

5) Let f be a scalar field and $\vec{\mathbf{F}}$ a vector field. State whether each expression is meaningful. If not, explain why. If so, state whether it is a scalar field or a vector field.

a) $\text{curl}\, f$
b) $\text{grad}\, f$
c) $\text{div}\, \vec{\mathbf{F}}$
d) $\text{curl}\left(\text{grad}\, \vec{\mathbf{F}}\right)$
e) $\text{grad}\, \vec{\mathbf{F}}$
f) $\text{grad}\left(\text{div}\, \vec{\mathbf{F}}\right)$
g) $\text{div}\left(\text{grad}\, f\right)$
h) $\text{grad}\left(\text{div}\, f\right)$
i) $\text{curl}\left(\text{curl}\, \vec{\mathbf{F}}\right)$
j) $\text{div}\left(\text{div}\, \vec{\mathbf{F}}\right)$
k) $\left(\text{grad}\, f\right)\times\left(\text{div}\, \vec{\mathbf{F}}\right)$
l) $\text{div}\left(\text{curl}\left(\text{grad}\, f\right)\right)$

a) Meaningless because f is a scalar field.
b) Vector Field.
c) Scalar Field.
d) Vector Field.
e) Meaningless because $\vec{\mathbf{F}}$ is not a scalar field.
f) Vector Field.
g) Scalar Field.
h) Meaningless because f is a scalar field.
i) Vector Field.
j) Meaningless because $\vec{\mathbf{F}}$ is not a scalar field.
k) Meaningless because $\vec{\mathbf{F}}$ is not a scalar field.
l) Scalar Field.

6) Determine whether or not the vector field $\vec{\mathbf{F}}(x, y, z) = ye^{-x}\,\mathbf{i} + e^{-x}\,\mathbf{j} + 2z\,\mathbf{k}$ is conservative.

Not Conservative

7) Find the curl and the divergence of the vector field $\vec{\mathbf{F}}(x, y, z) = \left\langle xe^{-y},\ xz,\ ze^{y} \right\rangle$.

$$\boxed{\text{curl } \vec{\mathbf{F}} = \left\langle ze^{y} - x,\ 0,\ z + xe^{-y} \right\rangle, \quad \text{div } \vec{\mathbf{F}} = e^{y} + e^{-y}}$$