

1) Find the length of the curve.

a) $\mathbf{r}(t) = \langle 2\sin t, 5t, 2\cos t \rangle, -10 \leq t \leq 10$

b) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}, 0 \leq t \leq 1$

a) $\boxed{20\sqrt{29}}$

b) $\boxed{e - e^{-1}}$

2) Reparametrize the curve with respect to arc length measured from the point where $t=0$ in the direction of increasing t .

a) $\mathbf{r}(t) = \langle 2t, 1-3t, 5+4t \rangle,$

b) $\mathbf{r}(t) = 3\sin t\mathbf{i} + 4t\mathbf{j} + 3\cos t\mathbf{k}$

a) $\boxed{\mathbf{r}(t(s)) = \left\langle \frac{2}{\sqrt{29}}, 1 - \frac{3}{\sqrt{29}}s, 5 + \frac{4}{\sqrt{29}}s \right\rangle}$

b) $\boxed{\mathbf{r}(t(s)) = \left\langle 3\sin\left(\frac{1}{5}s\right), \frac{4}{5}s, 3\cos\left(\frac{1}{5}s\right) \right\rangle}$

3) Find the curvature of $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.

$$\boxed{\kappa(1) = \frac{1}{7}\sqrt{\frac{19}{14}}}$$

4) Find the curvature of the following functions:

a) $y = x^3$

b) $y = \cos x$

a) $\frac{6|x|}{(1+9x^4)^{3/2}}$

b) $\frac{|\cos x|}{(1+\sin^2 x)^{3/2}}$

5) Given $\mathbf{r}(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$ find the vectors \mathbf{T} , \mathbf{N} , and \mathbf{B} at the point $\left(1, \frac{2}{3}, 1\right)$

$$\mathbf{T}(1) = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle, \mathbf{N}(1) = \left\langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle, \mathbf{B}(1) = \left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$$

6) Find the circle of curvature to the graph $y = e^x$ at $x = 0$.

$$(x+2)^2 + (y-3)^2 = 8$$